

Oscillations

(displacement, velocity, acceleration and equations of motion)

Note : This PPT will NOT help you learn physics concepts. It is intended only as a quick revision of formulas, definitions, theorems and concepts before examinations. No physics can be learnt just by watching a few videos or going through a few slides of PPT.



Simple harmonic motion (S H M)

A body is said to be executing SHM if its motion satisfies the following conditions

1. The motion is periodic
2. Motion is to and fro about a mean position
3. Acceleration is proportional to displacement
4. Acceleration and displacement are in opposite directions

Examples

1. Oscillations of a load attached to a spring
2. Oscillations of bob of a simple pendulum
3. Oscillations of a magnetic needle

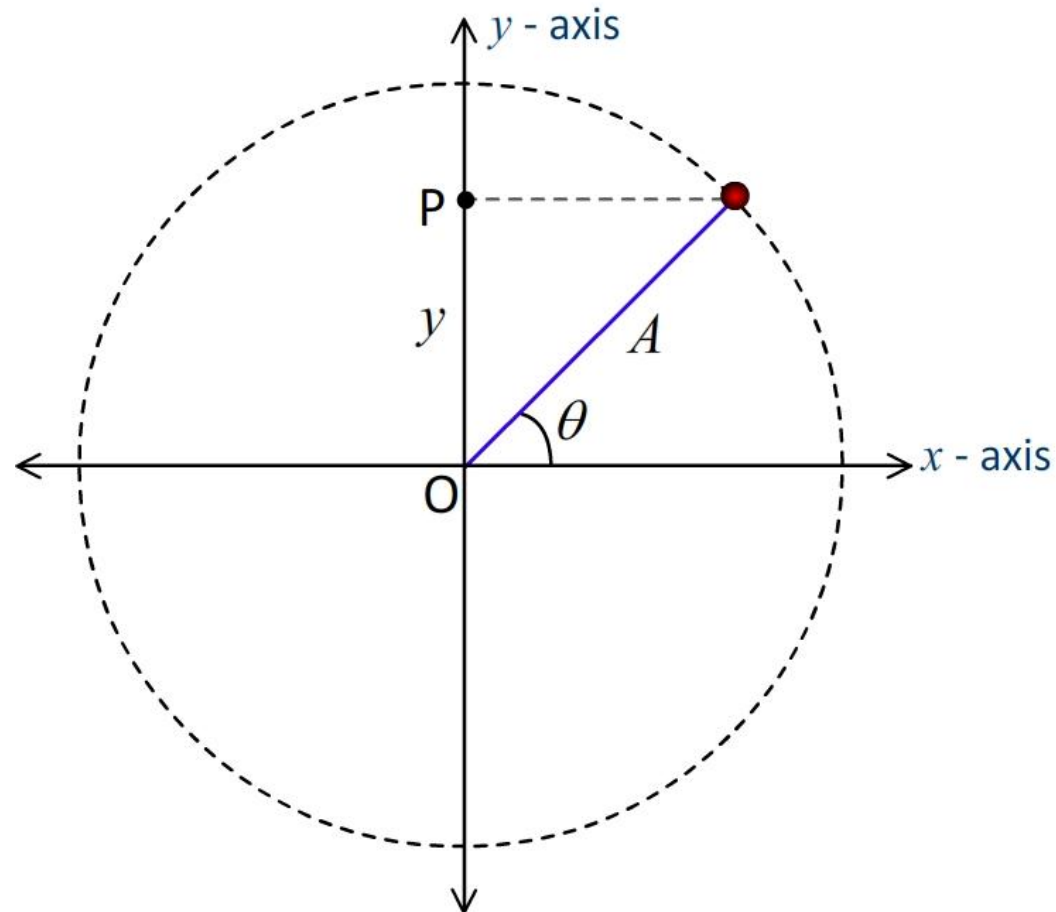
Reference circle (for analyzing SHM)

Consider a body executing uniform circular motion of radius A . Let ω be the uniform angular velocity of the body.

Let P be the foot of perpendicular drawn from the body to the y -axis.

As the body executes circular motion, the point P executes to-and-fro, periodic oscillatory motion about the mean position (i.e. the centre of the circle).

Let y be the instantaneous displacement of P from the origin at an instant of time (t).



Reference circle simulation

Reference circle (for analyzing SHM)

Displacement (y) of P from O is given by

$$y = A \sin(\theta) \text{ — i}$$

Using the relation $\theta = \omega t$ we get

$$y = A \sin(\omega t)$$

Eq (i) gives displacement of point P as a function of time .

Differentiating eq (i) w.r.t. time we get

$$\frac{dy}{dt} = \frac{dA \sin(\omega t)}{dt}$$
$$v = A\omega \cos(\omega t) \text{ — ii}$$

Eq (ii) gives velocity of point P as a function of time .

Differentiating eq (ii) w.r.t. time we get

$$\frac{dv}{dt} = \frac{d A\omega \cos(\omega t)}{dt}$$
$$a = -A\omega^2 \sin(\omega t) \text{ — iii}$$

Eq (iii) gives acceleration of point P as a function of time .

Using eq (i) we get

$$a = -\omega^2 y \text{ — iv}$$

From above equation it is observed that

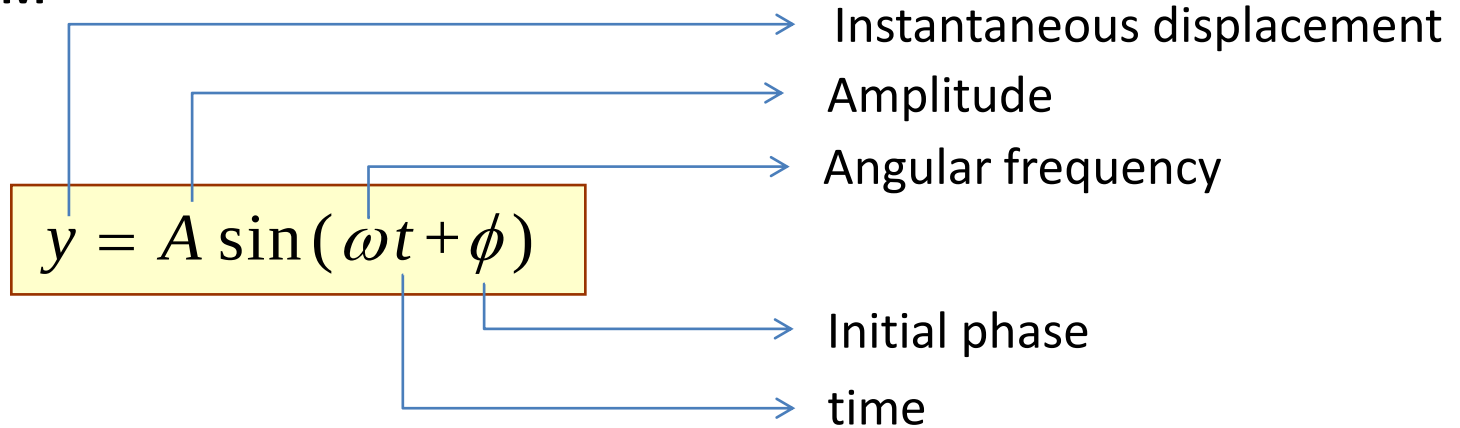
(i) $a \propto x$

(ii) a is in a direction opposite to x

therefore we can say that the point P executes SHM.

Oscillations

Parameters of SHM



Amplitude (A) : It is the maximum displacement from mean position

Frequency (n) : It is the number of oscillations completed in a unit interval of time

Time period (T) : It is the time taken for one complete oscillation

Initial phase (ϕ) : It is a parameter that determines the initial state of oscillation

Phase ($\omega t \pm \phi$) : It determines the instantaneous position and velocity of the body

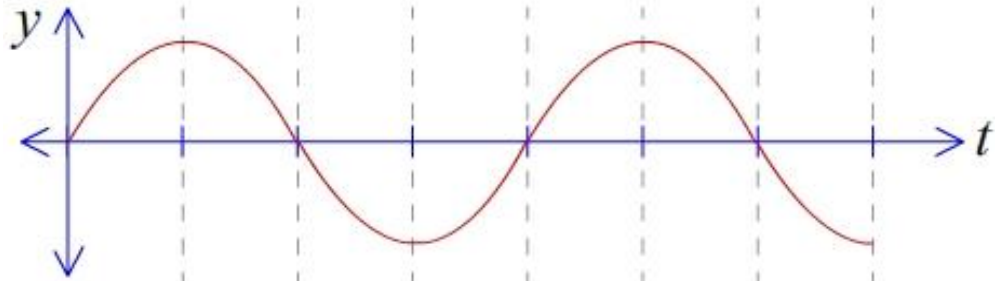
$$\omega = 2\pi n \quad n = \frac{1}{T}$$

Oscillations

Equations of motion (and their graphical representations) in SHM

Displacement-time graph

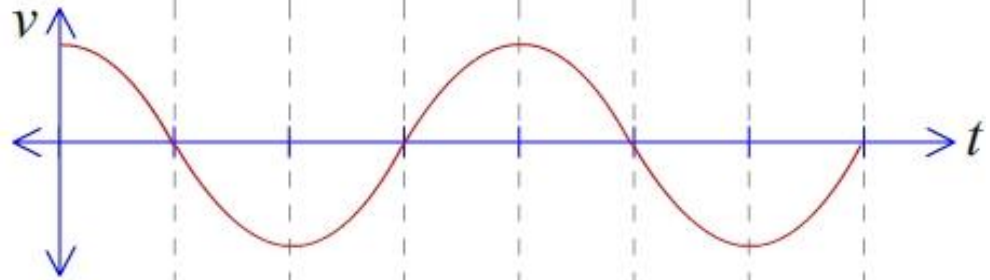
$$y = A \sin(\omega t)$$



Velocity- time graph

$$v = A\omega \cos(\omega t)$$

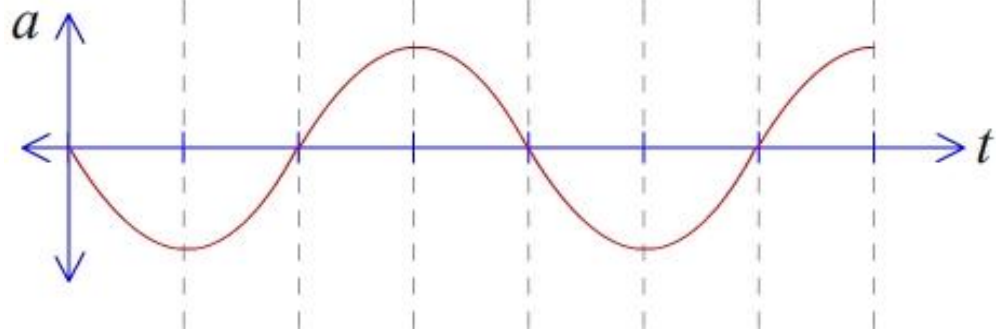
$$v = A\omega \sin(\omega t + \pi / 2)$$



Acceleration-time graph

$$a = -A\omega^2 \sin(\omega t)$$

$$a = A\omega^2 \sin(\omega t + \pi)$$



Parameters simulation

Velocity as a function of displacement

Velocity as a function of time is given by the relation

$$v = A\omega \cos(\omega t) \quad \text{--- i}$$

Using the relation

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\Rightarrow \sin^2(\omega t) + \cos^2(\omega t) = 1$$

$$\Rightarrow \cos^2(\omega t) = 1 - \sin^2(\omega t)$$

$$\Rightarrow \cos(\omega t) = \sqrt{1 - \sin^2(\omega t)} \quad \text{--- ii}$$

Substituting this in eq (i) we get

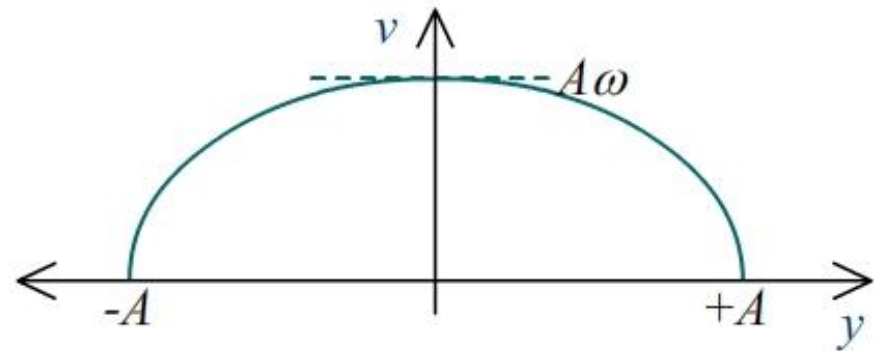
$$v = A\omega \sqrt{1 - \sin^2(\omega t)}$$

$$v = \omega \sqrt{A^2 - A^2 \sin^2(\omega t)}$$

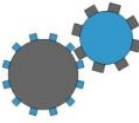
Using $y = A \sin(\omega t)$ we get

$$v = \omega \sqrt{A^2 - y^2}$$

Graph of velocity as a function of displacement is an ellipse



Velocity is maximum ($A\omega$) at the mean position ($y = 0$) and zero at extreme positions ($\pm A$).

  
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